Name: ____________________

Warm Up:

1) $f(x) = 2x + 1$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 2} f(x) =$

$f(2) =$

2) $f(x) = \dfrac{x^2 - 1}{x - 1}$

$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1^-} f(x) =$

$\lim_{x \to 1} f(x) =$

$f(1) =$

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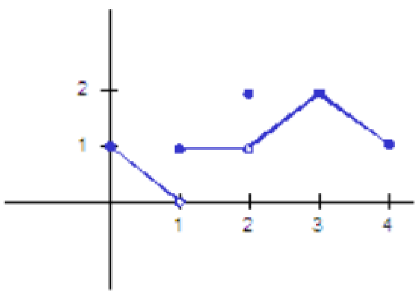
3)



a)

$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1^-} f(x) =$

$f(1) =$

b)

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

$f(2) =$

c)

$\lim_{x \to 3^+} f(x) =$

$\lim_{x \to 3^-} f(x) =$

$f(3) =$

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4) $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, \text{ if } & x \neq 2 \\ 7 & \text{if } x = 2 \end{cases}$

$f(2) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

5) $f(x) = \begin{cases} 3 + x^2, \text{ if } & x < 2 \\ 9 - x^2, \text{ if } & x \geq 2 \end{cases}$

$f(2) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 2^-} f(x) =$

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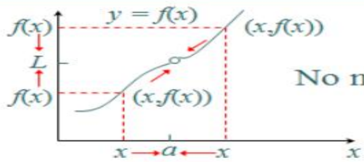
Definition of Limit of a Function

Suppose that the function $f(x)$ is defined for all values of x near a, but not necessarily at a. If as x approaches a (without actually attaining the value a), $f(x)$ approaches the number L, then we say that L is the limit of $f(x)$ as x approaches a, and write

$$\lim_{x \to a} f(x) = L$$



No matter how x approaches a, $f(x)$ approaches L.

<u>Properties of Limits</u>

1) The limit of a constant is that constant.
2) The limit as $x \to a$ of x is a.

3) The limit of a constant multiplied by a variable is equal to the constant times the limit of the variable.

$$\lim_{x \to a} cx = c \lim_{x \to a} x$$

4) The limit of a sum is equal to the sum of the limits. $\lim_{x \to a} [f(x) + g(x)] =$

5) The limit of a difference is equal to the difference of the limits. $\lim_{x \to a} [f(x) - g(x)] =$

6) The limit of a product is equal to the product of the limits. $\lim_{x \to a} [f(x) \bullet g(x)] =$

7) The limit of a quotient is equal to the quotient of the limits. $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] =$ If $\lim_{x \to a} g(x) \neq 0$

8) The limit of a function raised to a power is equal to the limit of the function and then raising it to that power.

$$\lim_{x \to a} [f(x)]^n =$$

9) The limit of the root of a function is equal to the root of the limit of the function.

$$\lim_{x \to a} \sqrt[n]{f(x)} =$$

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Examples:

1) $\lim_{x\to 4} 7 =$ ____

2) $\lim_{x\to 2} x =$ ____

3) $\lim_{x\to 4} 3x =$

4) $3\lim_{x\to 4} x =$ ____

5) $\lim_{x\to 5}\left(x^4 - x^2\right) =$ ________

6) $\lim_{x\to 3}(x+4)(x-1) =$

7) $\lim_{x\to 6}\left(\dfrac{x+12}{x}\right) =$

8) $\lim_{x\to 4}(x+2)^2 = \left[\lim_{x\to 4}(x+2)\right]^2$

9) $\lim_{x\to 4}\sqrt{25x} =$

Example:

GIVEN: $\lim_{x\to a} f(x) = -1$ $\lim_{x\to a} g(x) = 12$ $\lim_{x\to a} h(x) = 0$

Using your limit properties, find the limit if it exists.

1. $\lim_{x\to a}\left[f(x)+h(x)\right] =$

2. $\lim_{x\to a}\dfrac{1}{f(x)} =$

3. $\lim_{x\to a}\left[f(x)\right]^2 =$

4. $\lim_{x\to a}\sqrt[2]{g(x)} =$

5. $\lim_{x\to a}\left(\dfrac{4h(x)-f(x)}{g(x)}\right) =$

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Unit 2: Methods for Finding Limits

Method #1: Apply direct substitution

Examples:

$\lim_{x \to 4}(x^2 + 3x + 5) =$

$\lim_{x \to -1}\left(\frac{x}{x^3 + 8}\right) =$

$\lim_{x \to \pi} \cos x =$

Method #2: Algebraic simplification and then direct substitution.

Examples:

$\lim_{x \to 3}\left(\frac{x^2 - 2x - 3}{x - 3}\right) =$

$\lim_{x \to -5}\left(\frac{4x^2 + 19x - 5}{2x + 10}\right) =$

$\lim_{x \to 0}\left(\frac{x^2 + 5x}{x^3 - 25x}\right) =$

NOTE:____

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Unit 2: Other Methods for Finding Limits

Some equations can be more difficult to determine the limit.
For this type of problem you may look to use the following options!

3. Table of Values → **(Use TABLESET and TABLE options on your graphing Calculator!)**

Examples:

$$\lim_{x\to 1}\left(\frac{x^4-1}{x-1}\right)=$$

X	$\frac{x^4-1}{x-1}$
0.96	
0.97	
0.98	
0.99	

X	$\frac{x^4-1}{x-1}$
1.04	
1.03	
1.02	
1.01	

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=$$

X	$\frac{\sin x}{x}$
-0.04	
-0.03	
-0.02	
-0.01	

X	$\frac{\sin x}{x}$
0.04	
0.03	
0.02	
0.01	

$$\lim_{x\to 3}\left(\frac{x^3-27}{x-3}\right)=$$

X	$\left(\frac{x^3-27}{x-3}\right)$

X	$\left(\frac{x^3-27}{x-3}\right)$

Viewing the graph of the equation can also help determine the limit.

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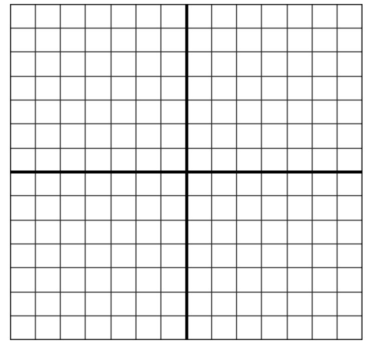
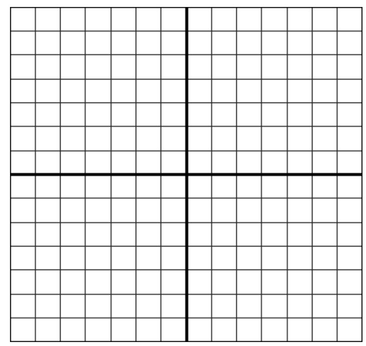
4. Graphing → **(Set the appropriate WINDOW and use TRACE key.)**

Examples:

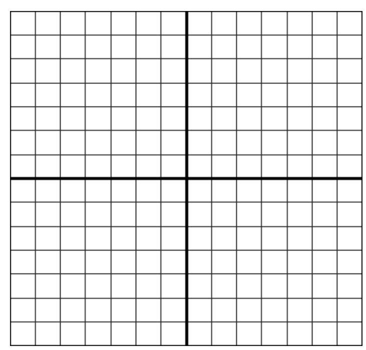
$$\lim_{x\to 0}\left(\frac{\sin x}{3x}\right) =$$



$$\lim_{x\to 1}\frac{|x-1|}{x-1} =$$



$$\lim_{x\to 1}\frac{x^3-1}{x-1} =$$



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Unit 2 Worksheet 2

AP Calculus AB

Evaluate the following limits.

1. $\lim_{x\to 8} 7 =$

2. $\lim_{x\to 0} \pi =$

3. $\lim_{x\to 3}(7x-4) =$

4. $\lim_{x\to -1}(2x^3-5x) =$

5. $\lim_{x\to 5}\sqrt{x^3-3x-10} =$

6. $\lim_{x\to -3}\sqrt{5x^2+3x} =$

7. $\lim_{x\to 3}\dfrac{x^2-2x}{x+1} =$

8. $\lim_{x\to 0}\dfrac{6x-9}{x^3-12x+3} =$

9. $\lim_{x\to 4}\dfrac{x^2+2x-24}{x-4} =$

10. $\lim_{x\to -2}\dfrac{x^2+7x+10}{x+2} =$

11. $\lim_{x\to 4}\dfrac{x^2-16}{x-4} =$

12. $\lim_{x\to -1}\dfrac{x^2+7x+6}{x^2-4x-5} =$

13. $\lim_{x\to 2}\dfrac{x^2-2x}{x^2-4} =$

14. $\lim_{t\to 9}\dfrac{9-t}{3-\sqrt{t}} =$

15. $\lim_{t\to 4}\dfrac{4-t}{2-\sqrt{t}} =$

16. $\lim_{x\to \frac{\pi}{2}} \sin x =$

17. $\lim_{x\to \frac{\pi}{2}} \cos x =$

18. $\lim_{x\to \frac{3\pi}{2}} \sin x =$

19. $\lim_{x\to 2\pi} \cos x =$

20. $\lim_{x\to 2}\dfrac{x^2-4x+4}{x^2+x-6} =$

21. $\lim_{x\to 1}\dfrac{1-x}{x^2-1} =$

22. $\lim_{x\to 5}\dfrac{5-x}{x^2-25} =$

23. $\lim_{x\to 0}\dfrac{x}{x} =$

24. $\lim_{x\to 3}\dfrac{x-3}{6-2x} =$